

EQUULEUS' In-Space Least Squares Thrust Vector Estimation with Non-linear Observation Model

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(Received May 29th, 2023)

This study presents a novel in-space thrust estimation algorithm for the EQUULEUS spacecraft, designed to address the challenges encountered during its initial operation prior to the first lunar gravity assist. With limited communication opportunities and a tight time window, traditional ground-based thrust estimation techniques are not viable. To overcome this, an in-space thrust estimation approach is adopted. The proposed short-term estimation algorithm accurately estimates the thrust and center of gravity-to-thruster vectors based on Doppler measurement data and changes in angular momentum over multiple observations. The mathematical derivations for the observation model proposed are validated through numerical simulations. The use of covariance as a metric for observation attitude selection has been shown to be very efficient in ensuring thrust vector estimation accuracy. Following the successful launch of the EQUULEUS spacecraft in November 2022, the proposed algorithm was applied to the real scenario. The outcomes from satellite operations are presented and analysed. The results of this research contribute to the field of spacecraft operations by providing advanced in-space thrust estimation techniques that can be utilized in future deep-space missions, ultimately enhancing the reliability of space exploration endeavors.

Key Words: In-space Thrust Vector Estimation, Attitude Control, Delta-V, Deep Space Cubesat, EQUULEUS

Nomenclature

$\Delta \mathbf{p}_{DV}$	: impulse vector by ΔV thruster, Ns
$r_{DV,i}$	: i th c.g.-to-thruster component, m
$\mathbf{q}_{ib}$	: quaternion from inertial to body frame
$\mathbf{v}_E$	: velocity in Earth-direction, m/s
$\mathbf{H}$	: Angular momentum, Nms
v_{acc}	: observation error of $\Delta \mathbf{v}_E$, m/s
$\mathbf{v}_{torque}$	: observation error of $\Delta \mathbf{H}$, Nms
$\hat{\mathbf{r}}_{sc}^{(i)}$	: unit vector from Earth to spacecraft
m_{sc}	: mass of spacecraft, kg
$\mathbf{k}$	: vector to spacecraft with norm of m_{sc}^{-1}
$\bar{\Delta \mathbf{p}}_{DV}$	: nominal impulse vector, Ns
$\bar{r}_{DV,i}$	: nominal c.g.-to-thruster component, m
σ_z^2	: variance of observation noise
P_x	: covariance matrix of x
$\Delta \alpha$	: deviation angle, deg
DVT1	: delta-v thruster 1
DVT2	: delta-v thruster 2
Subscripts	
i	: component of index i
ii	: matrix element row i , column i
Superscripts	
(i)	: i th observation

1. Introduction

In recent years, the revolution of space exploration has been greatly propelled by the emergence of nanosatellites, particularly Cubesats. These small spacecrafts have enabled cost-effective and agile missions, primarily focused on near-Earth applications. However, the success of these missions has laid

the foundation for venturing into more ambitious deep-space missions.¹⁾

Deep-space missions present unique challenges that differ from their near-Earth counterparts. In addition to the vast distances and extended mission durations, deep-space missions often encounter limited communication opportunities and face constrained time windows for crucial operations. These challenges require the development of specialized techniques and algorithms to ensure mission success and reliability. This is also the case in the EQUULEUS (Equilibrium Lunar-Earth point 6U Spacecraft) mission.

EQUULEUS, a deep space CubeSat developed by The University of Tokyo and the Japan Aerospace Exploration Agency (JAXA), embarked on its mission during the Artemis 1 launch in November 2022. The primary objective of EQUULEUS is to explore the Earth-Moon Lagrange point (EML-2) and demonstrate low-energy trajectory-control techniques within the Sun-Earth-Moon region. To achieve this, the spacecraft relies on gravity assists from celestial bodies and employs the water resistojet propulsion system, known as AQUARIUS. By leveraging these methods, EQUULEUS aims to showcase the effectiveness of novel water-based propulsion in orbital transfers, ultimately reducing fuel consumption and highlighting the potential of this technology for future space missions.^{2,4)}

However, in order to ensure the safe and successful execution of its mission, EQUULEUS must overcome various challenges. One significant concern is the performance of the propulsion system during the first delta-V operation (DV1) preceding the lunar swing-by. Several factors contribute to this concern. Firstly, the mission timeline and limited communication opportunities before the first lunar gravity assist make traditional Doppler-based estimation methods using deep-space networks

(DSN) unreliable. Additionally, the lack of heritage and knowledge regarding thruster performance in orbital transfers further compounds the challenge.

To address these challenges and ensure the precise maneuvering of the spacecraft, this study proposes an in-space thrust estimation approach. Building upon the previous work conducted by Tomita et al.,⁵⁾ the proposed algorithm utilizes Doppler measurement data and analyzes changes in angular momentum observed over multiple instances. By doing so, it provides accurate short-term thrust and center of gravity-to-thruster vector estimates. Leveraging in-space observations, the algorithm enhances the accuracy and reliability of thrust estimation, specifically tailored to the unique challenges encountered by the EQUULEUS spacecraft.

Following the successful launch of the EQUULEUS spacecraft and the implementation of the proposed algorithm, the real-world outcomes from satellite operations are also presented and analyzed. The results shared in this study contribute to the field of spacecraft operations by providing advanced in-space thrust estimation techniques that can be applied to future deep-space missions.

The paper is structured as follows: Section 2. provides an overview of the observation strategy and observation model, explaining the methodology employed to acquire the necessary data for thrust estimation. In Sec. 3., the process of linearizing the model is presented, which enables the development of an estimation algorithm. The least squares estimation technique used to accurately estimate thrust and center of gravity-to-thruster vectors is described in Sec. 4.. The issue of state observability, crucial for the algorithm's performance, is examined in Sec. 5.. Section 6. delves into the attitude selection strategy, which utilizes covariance as a metric to ensure precise thrust vector estimation. The validity of the proposed algorithm and observation model is assessed through simulations in Sec. 7. and real operational scenarios in Sec. 8.. Section 9. offers a comprehensive discussion and analysis of the obtained results. Finally, in Sec. 10., the conclusions drawn from the study are summarized, and potential future developments in the field are outlined.

2. Observation Strategy & Observation Model

In the previous model,⁵⁾ by combining angular momentum variations and Doppler measurements it was possible to estimate the magnitude of the thrust vector for each thruster. However, for successful thruster calibration and accurate execution of the ΔV maneuver, thrust vector direction for each thruster and their relative positions to the center of gravity are also required. For that reason, a new set of state variables were adopted.

For a single thruster, let $\mathbf{r}_{DV}$ be the vector from the center of gravity to the thruster location in body-fixed coordinate frame. Let also $r_{DV,1}$, $r_{DV,2}$, and $r_{DV,3}$ represent its x, y and z components respectively. The state vector for a single thruster is then defined as

$$\mathbf{x} \equiv \begin{bmatrix} \Delta \mathbf{p}_{DV} \\ r_{DV,1} \\ r_{DV,2} \end{bmatrix}, \quad (1)$$

where $\Delta \mathbf{p}_{DV}$ is the impulse vector by the thruster in body-fixed

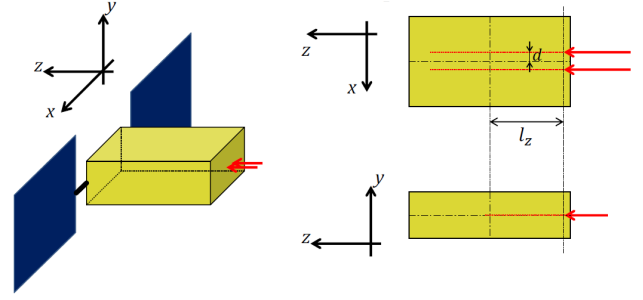


Fig. 1. Layout of delta-V thrusters in EQUULEUS. The red arrows represent the thrust vectors.⁵⁾

coordinate frame. Since the impulse is the integral of thrust over time, the impulse vector represents the total executed thrust vector in each activation. Moreover, the z component $r_{DV,3}$ is considered constant due to the structure configuration of the spacecraft and will be further referred in the text as the sixth state variable $\bar{x}_6 = l_z$ as in Fig. 1.

The relationship between the observation variables, specifically variations in velocity and angular momentum, and the defined state vector can be directly obtained through physics. The Doppler effect enables velocity shift (ΔV) measurements, while the changes in the rotation speed of reaction wheels are directly related to changes in angular momentum via the spacecraft's moment of inertia. To obtain these measurements, we consider the following observation strategy for the i-th measurement:

1. Fix the initial attitude position at $\mathbf{q}_{i2b}^{(i)}$;
2. Perform Doppler and total angular momentum measurements
3. Activate one thruster for a specified time $t^{(i)}$;
4. Perform Doppler and total angular momentum measurements again;
5. Compute the variation of the Earth direction-component of velocity, denoted as $\Delta \mathbf{v}_E^{(i)}$, and change in angular momentum, denoted as $\Delta \mathbf{H}^{(i)}$, in the body-fixed coordinate frame.

The observation model than is defined for the i-th observation as:

$$\mathbf{z}^{(i)} = \begin{bmatrix} \Delta \mathbf{v}_E^{(i)} \\ \Delta \mathbf{H}^{(i)} \end{bmatrix} + \mathbf{v}^{(i)}, \quad (2)$$

in which $\mathbf{v}^{(i)}$ represents the associated measurement errors.

To adequate the observation model to the defined state vector, consider each observation variable separately. For the i-th Doppler measurement we have:

$$\begin{aligned} z_{\text{Doppler}}^{(i)} &= \Delta \mathbf{v}_E^{(i)} + v_{\text{acc}}^{(i)} \\ &= \hat{\mathbf{r}}_{sc}^{(i)T} \frac{\Delta \mathbf{p}_{DV}}{m_{sc}^{(i)}} + v_{\text{acc}}^{(i)}, \end{aligned} \quad (3)$$

in which $v_{\text{acc}}^{(i)}$ is the observation error in line-of-sight acceleration computed by Doppler measurements, $\hat{\mathbf{r}}_{sc}^{(i)}$ is the direction vector from Earth to the spacecraft's center of gravity, and m_{sc} is the spacecraft's mass. For this analysis, it is considered that the mass is constant and that, given the dimensions of the spacecraft, its center of gravity and center of mass coincide.

For the change in angular momentum in each axis, the measurement equation is as follows:

$$\begin{aligned} z_{\text{AM}}^{(i)} &= \Delta \mathbf{H}^{(i)} + \mathbf{v}_{\text{torque}}^{(i)} \\ &= \mathbf{r}_{DV} \times \Delta \mathbf{p}_{DV} + \mathbf{v}_{\text{torque}}^{(i)}, \end{aligned} \quad (4)$$

where $\mathbf{v}_{\text{torque}}^{(i)}$ is the observation noise in the ΔV thrust calculation by reaction wheel rotation speed measurements.

Then, the observation model for the i -th observation can be improved by relating the defined model in Eq. (2) to the state vector $\mathbf{x}$ defined in Eq. (1) via the relationships derived in Eqs. (3) and (4). Let $\mathbf{k} \equiv \hat{\mathbf{r}}_{sc}^{(i)}/m_{sc}$, we can write:

$$\mathbf{z}^{(i)} = f(\mathbf{k}^{(i)}, \mathbf{x}) + \mathbf{v}^{(i)}, \quad (5)$$

with

$$f(\mathbf{k}^{(i)}, \mathbf{x}) = \begin{bmatrix} k_1 x_1 + k_2 x_2 + k_3 x_3 \\ x_5 x_3 - \bar{x}_6 x_2 \\ \bar{x}_6 x_1 - x_4 x_3 \\ x_4 x_2 - x_5 x_1 \end{bmatrix}, \quad (6)$$

and

$$\mathbf{v}^{(i)} = \begin{bmatrix} v_{\text{acc}}^{(i)} \\ v_{\text{torque}}^{(i)} \end{bmatrix}. \quad (7)$$

Finally, when n observations are considered the model is given as follows:

$$\begin{bmatrix} \mathbf{z}^{(1)} \\ \vdots \\ \mathbf{z}^{(n)} \end{bmatrix} = \begin{bmatrix} f(\mathbf{k}^{(1)}, \mathbf{x}) \\ \vdots \\ f(\mathbf{k}^{(n)}, \mathbf{x}) \end{bmatrix} + \begin{bmatrix} \mathbf{v}^{(1)} \\ \vdots \\ \mathbf{v}^{(n)} \end{bmatrix}. \quad (8)$$

3. Model Linearization

The model described by Eqs. (5) and (8) is non-linear which is made apparent by the presence of cross-terms of the state variables. However, it is possible to obtain a linearized model at nominal regime. Let us define the single-thruster nominal values for the state vector $\mathbf{x}$ as:

$$\bar{\mathbf{x}} = \begin{bmatrix} \bar{\Delta p}_{DV} \\ \bar{r}_{DV,1} \\ \bar{r}_{DV,2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \pm d \end{bmatrix}, \quad (9)$$

where $\bar{\Delta p}_{DV}$ represents the nominal impulse, and d is the displacement in the x -axis from the y - z plane (positive for one thruster and negative for the other). The nominal observed variable $\bar{\mathbf{z}}^{(i)}$ can then be defined as

$$\bar{\mathbf{z}}^{(i)} = f(\mathbf{k}^{(i)}, \bar{\mathbf{x}}). \quad (10)$$

Finally, the linearized observation model can be expressed as

$$\delta \mathbf{z} \equiv \begin{bmatrix} \delta \mathbf{z}^{(1)} \\ \vdots \\ \delta \mathbf{z}^{(n)} \end{bmatrix} = \begin{bmatrix} \frac{\partial f(\mathbf{k}^{(1)}, \mathbf{x})}{\partial \mathbf{x}} \big|_{\mathbf{x}=\bar{\mathbf{x}}} \\ \vdots \\ \frac{\partial f(\mathbf{k}^{(n)}, \mathbf{x})}{\partial \mathbf{x}} \big|_{\mathbf{x}=\bar{\mathbf{x}}} \end{bmatrix} \delta \mathbf{x} + \begin{bmatrix} \mathbf{v}^{(1)} \\ \vdots \\ \mathbf{v}^{(n)} \end{bmatrix} \equiv A \delta \mathbf{x} + \mathbf{v}, \quad (11)$$

in which $\delta \mathbf{z} = \mathbf{z} - \bar{\mathbf{z}}$, $\delta \mathbf{x} = \mathbf{x} - \bar{\mathbf{x}}$ and

$$\frac{\partial f(\mathbf{k}^{(i)}, \mathbf{x})}{\partial \mathbf{x}} = \begin{bmatrix} k_1^{(i)} & k_2^{(i)} & k_3^{(i)} & 0 & 0 \\ 0 & -\bar{x}_6 & x_5 & 0 & x_3 \\ \bar{x}_6 & 0 & -x_4 & -x_3 & 0 \\ -x_5 & x_4 & 0 & x_2 & -x_1 \end{bmatrix}. \quad (12)$$

4. Least Squares Estimation

In Section 3, a linearized model was derived. Since different types of data are measured by the considered observation method, it is common to introduce some sort of normalization prior to estimation. The normalization factor can be obtained by whitening the observation errors.³⁾ Consider that the observation error $\mathbf{v}$ has zero mean. The error covariance matrix is given by:

$$E[\mathbf{v}\mathbf{v}^T] = \begin{bmatrix} \sigma_{z_1}^2 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \sigma_{z_n}^2 \end{bmatrix}, \quad (13)$$

in which $E[\cdot]$ is the expected value operator. Consider then the matrix R which satisfies:

$$RE[\mathbf{v}\mathbf{v}^T]R^T = I, \quad (14)$$

with I as the identity matrix. Then, one can verify that R is given by

$$R = \begin{bmatrix} \sigma_{z_1}^{-1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \sigma_{z_n}^{-1} \end{bmatrix}. \quad (15)$$

With the matrix R obtained in Eq. (15), the normalized linear model is given by:

$$\delta \mathbf{z}' = A' \delta \mathbf{x} + \mathbf{v}', \quad (16)$$

in which $A' = RA$, $\delta \mathbf{z}' = R\delta \mathbf{z}$, and $\mathbf{v}' = R\mathbf{v}$.

Finally, state estimation for the normalized system described by Eq. (16) can be obtained from the least squares solution as shown in Eq.(17).

$$\delta \hat{\mathbf{x}} = (A'^T A')^{-1} A'^T \delta \mathbf{z}'. \quad (17)$$

A major advantage of the adopted normalization method is that the estimate state error covariance P_x can be directly obtained by the expression in Eq. (18).³⁾ The importance of this covariance matrix will be discussed on the following sections.

$$P_x = E[(\delta \mathbf{x} - \delta \hat{\mathbf{x}})(\delta \mathbf{x} - \delta \hat{\mathbf{x}})^T] = (A'^T A')^{-1}. \quad (18)$$

5. State Observability

Equation (18) presents a very convenient way to obtain the estimate state error covariance. However, for this relation to be valid it is necessary that the matrix $A^T A$ is fully observable, that is, $\text{rank}(A^T A) = \dim(\delta \mathbf{x}) = 5$. Until this point, a n -observations model has been considered, but for simplicity let us examine the $n = 2$ case. As derived in the appendix, it can be shown that

$$\text{rank}(A^T A) = \text{rank} \begin{bmatrix} k_1^{(1)} & k_2^{(1)} & k_3^{(1)} & 0 & 0 \\ k_1^{(2)} & k_2^{(2)} & k_3^{(2)} & 0 & 0 \\ 0 & -\bar{x}_6 & 0 & 0 & \bar{\Delta p}_{DV} \\ \bar{x}_6 & 0 & \mp d & -\bar{\Delta p}_{DV} & 0 \\ 0 & \pm d & 0 & 0 & 0 \end{bmatrix}. \quad (19)$$

From Eq. (19) it can be seen that if $k^{(1)} \neq k^{(2)}$, then $\text{rank}(A^T A) = 5$ and the system is fully observable. However, if the two k vectors are the same, then $\text{rank}(A^T A) = 4 < \dim(\delta x) = 5$ and observability is not achieved. Therefore, at least two different k vector values are required, which means at least two different attitude positions must be selected for observation.

6. Attitude selection strategy

In order to successfully estimate the state variables, it was found that at least two different attitude positions are required. However, it is not yet been clarified how should one make such choice or its impacts to the estimation results. To address this issue the covariance matrix presented in Eq. (18) was selected as an evaluation tool.

The selection criteria should be such that the pair of attitudes considered for observation minimizes overall state estimation variance. In Eq. (18) let us suppress the variable index such that $P_x \equiv P$. With this notation, let P_{ii} be the i -th diagonal element in P . Then, the variance of each state variable x_i can be obtained from the corresponding P_{ii} . This way, given the A' matrix it is possible to calculate the state estimation variances and evaluate the selected attitudes.

6.1. EQUULEUS' Attitude Selection

In the case of EQUULEUS, the initial attitude was predetermined and set as the first ΔV operation (DV1) attitude before correction. Given the propulsion system's sensitivity to thermal variations in EQUULEUS, maintaining consistent thrust response across observations required selecting candidate attitudes that retain the body-frame sun direction of the initial attitude. Figure 2 presents a visualization of how attitude selection affects the covariance values of one state variable for any two candidate attitudes. It is noticeable that, in addition to values close to the main diagonal, which correspond to similar attitudes, two extra bright lines are also observed. These lines yield high covariance values and thus should be avoided.

Since the initial attitude was fixed to DV1, the covariance values of interest should be calculated with respect to that attitude, which is referred to as *local covariance* in this context. On the other hand, the more general case shown in Fig. 2 is referred to as *global covariance*, encompassing a broader perspective. Figure 3 displays the local covariance values for all estimated state variables. For each curve in Fig. 3, the existence of local minima and local maxima points is observed. During operation, the rotation angle should be set within the range of 90 to 210 degrees to mitigate potential large estimation errors. Specifically, for the case shown in Fig. 3, the ideal choice for the rotation angle would be around 150 degrees.

This example illustrates the utility of the covariance analysis to operators. Despite its simplicity, the visualization tool employed can provide valuable insights and facilitate the decision-making process.

7. Simulation Results

In this section, a verification case study was conducted in order to evaluate the efficacy of the proposed estimation algo-

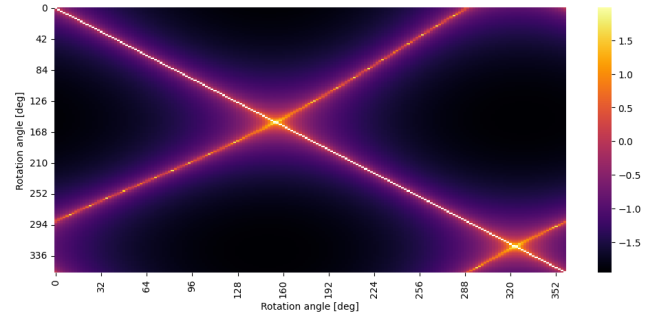


Fig. 2. Estimate state error global covariance P_{11} .

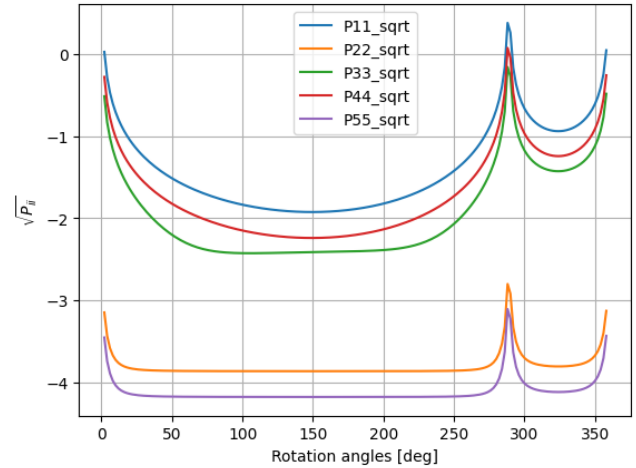


Fig. 3. Estimate state error local covariance.

rithm prior to launch. Here, the simulation results considering the proposed attitude selection strategy and thrust estimation algorithm are summarized.

The objective of this simulation study is to draw insightful conclusions about the proposed algorithm's performance by closely simulating satellite operations. The simulated data plays a crucial role in this evaluation. As the post-launch thrust vector estimation relies on telemetry data acquired through the proposed observation strategy, it is vital that the simulated data exhibits similar characteristics to effectively substitute for the actual telemetry data.

The simulation data was generated using a procedure similar to the one described by Tomita.⁵⁾ A time series of reaction wheel rotation speeds and Doppler measurements (residual o-c) was calculated based on an appropriate thrust profile derived from slightly modifying the original profile observed in ground experiments, and taking into account EQUULEUS' structural properties. These modifications, involving small tilts of the thrust vector with respect to the nominal direction, were made to better align with the observation objectives. The simulation data for the first attitude (DV1 attitude) is shown in Fig. 4.

The simulation scenario considered the initial observation time set to 00:00 UTC on September 8th, 2022. Furthermore, the number of activations for each thruster was set to 6 activations per attitude, which can be identified in Fig. 4 by the colored stripes. Based on each set of activations, the impulse estimation was conducted, and results for the first ΔV thruster (DVT1) are summarized in table 1.

As shown in table 1, there is a clear improvement in the estimation of the x and y direction components. Furthermore, con-

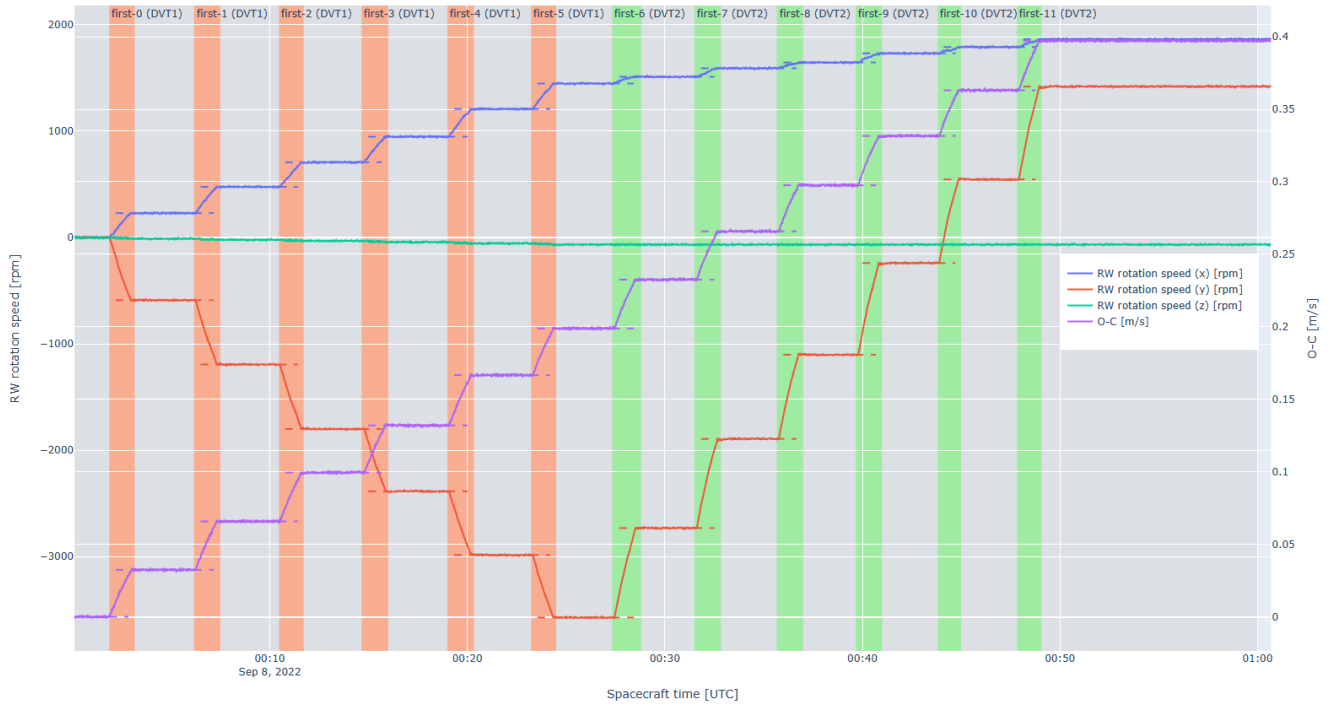


Fig. 4. Simulated reaction wheel rotation speeds and Doppler measurements (residual o-c) at the first attitude. Each colored rectangle correspond to a single-thruster activation as indicated by the corresponding name tags.

Table 1. Impulse vector (magnitude and unitary direction components), deviation angle ($\Delta\alpha$) and angular error for DVT1 obtained via simulations.

	True	Initial Estimate	Final Estimate
x -direction	-0.004	-0.020	-0.0038
y -direction	0.017	0.026	0.014
z -direction	0.999	0.999	0.999
$ \Delta p_{DV} [Ns]$	0.3821	0.3780	0.3785
$\Delta\alpha$ [deg]	1.00	1.88	0.83
Error [mdeg]	0	8.84	0.36

sidering that the previous algorithm already provided a good estimation of the magnitude of the impulse vector, while there is some improvement with the new algorithm, it may not be particularly remarkable. Figure 5 further highlights the improvements in direction estimation on the x-y plane.

From the operator's perspective, the crucial information extracted from the in-space thrust (or impulse) estimation is the deviation angle $\Delta\alpha$. In this context, the deviation angle is defined as the angular displacement between the nominal impulse direction vector and the executed one. This deviation value enables operators to make minor adjustments to the ΔV attitude, ensuring that the executed ΔV vector from each thruster closely matches the desired values. According to the results presented in table 1, the proposed method successfully reduced the angle error by an order of magnitude providing a good approximation of the real executed thrust vector.

Another important aspect of the proposed algorithm is the r_{DV} estimation capability. The estimated center of gravity-to-thruster location results are displayed in tab. 2 and Fig. 6. The results show a good agreement between the true and the estimated values, which further confirms the efficacy of the proposed algorithm.

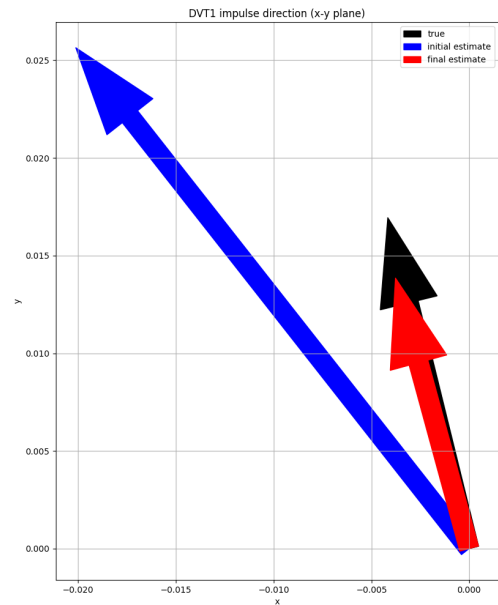


Fig. 5. Comparison of impulse unit direction components in x-y plane. Estimation with the previous algorithm is shown in blue, while estimation with the new algorithm is shown in red. Black represents the true value.

Table 2. True and estimated C.G. to thruster vector components for both ΔV thrusters. (*The sign of the nominal value is positive for DVT1 and negative for DVT2.)

	Nominal*	True 1	DVT1	True 2	DVT2
$r_{DV,1}$	± 0.0125	0.0119	0.0122	-0.0125	-0.0129
$r_{DV,2}$	0.0	0.0015	0.0016	0.0010	0.0014

8. Operation Results

On November 16th, 2022, the EQUULEUS spacecraft successfully launched on board of NASA's Space Launch System

(SLS) rocket. The in-space thrust estimation operation initiated at around 01:00 UTC on November 17th, 2022. Following the proposed observation strategy, the Doppler measurements and reaction wheel rotation speed data recorded in telemetry was successfully obtained. Figure 7 shows the measured values for the first observation attitude. In contrast to the simulation settings, the ADCS team operators made individual decisions on the activation intervals for each thruster. These decisions were based on a visual inspection of the data including some margin to completely encompass the relevant data, as indicated by the stripes in Fig. 7.

The second observation attitude followed the same process as the previous one. Based on the covariance results for the DV1 attitude, a rotation angle of 180 degrees was applied between the first and second observation attitudes. The resultant impulse values obtained from the estimation procedure are summarized in Table 3. Figure 8 illustrates the initial and final estimation vectors in the x-y plane for both thrusters (DVT1 and DVT2).

The center of gravity to thruster estimation results are summarized in Table 4 and displayed in Figure 9. Upon initial observation, a noticeable disparity becomes apparent when comparing the nominal vector in Figure 9 with Figure 6. This difference arises from a measured shift in the center of gravity along the x-axis during operations, prompting adjustments to the nominal values for enhanced accuracy. Additionally, Figure 8 reveals a discernible distinction between the initial and final estimations. This discrepancy can be attributed to the initial estimation's high sensitivity to changes in the center of gravity, which does not appear to be experienced by the proposed algorithm.

Table 3. Impulse vector (magnitude and unitary direction components) and deviation angle ($\Delta\alpha$) for DVT1 obtained during operation.

	Initial Estimate	Final Estimate
x-direction	-0.006	0.015
y-direction	0.004	0.006
z-direction	0.999	0.999
$ \Delta p_{DV} [Ns]$	0.5837	0.5840
$\Delta\alpha$ [deg]	0.39	0.91

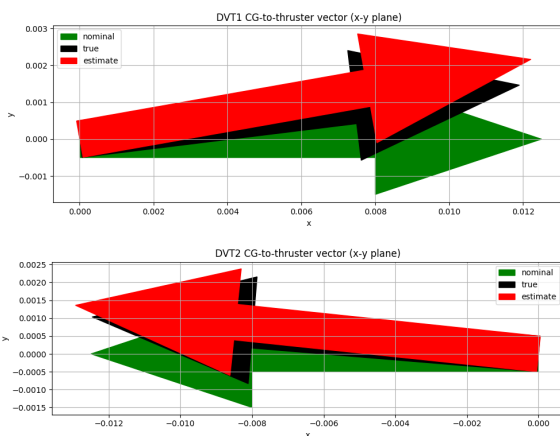


Fig. 6. Comparison between nominal, true and estimated center of gravity to thruster vector x-y components for both thrusters.

Table 4. Nominal and estimated C.G. to thruster vector components for both ΔV thrusters.

	Nominal 1	Nominal 2	DVT1	DVT2
$r_{DV,1}$	0.0152	-0.0068	0.01150	-0.0091
$r_{DV,2}$	0.00001	0.00001	-0.0005	-0.0012

9. Discussion

An unanswered aspect of utmost importance is the effectiveness of in-space thrust vector estimation during operation. Alternatively, we can explore the impact of the corrections made to each thruster on the overall ΔV execution error. Addressing these questions presents significant challenges for several reasons. Firstly, the lack of information regarding the executed thrust direction during DV1 from the perspective of orbit determination and trajectory control hinders the establishment of a reference point for comparing estimation results and total execution angle error. Secondly, the relatively low reliability of each thruster's executed thrust vector complicates the assessment of individual estimations and introduces complexities in identifying potential errors.

However, when considering DV2, which provides ample data for analysis, an investigation can be conducted to verify the accuracy of the executed thrust vector. Figure 10 illustrates the executed thrust vector during DV2, alongside the previously calculated thrust directions for DVT1 and DVT2. Based on these results, it becomes challenging to discern the individual contributions of each thruster in decreasing or potentially increasing the angular error.

On one hand, the close alignment between the executed thrust direction and the direction of the adjustment introduced to reduce misalignment between the nominal and estimated thrust direction raises concerns about the effectiveness of the correction performed based on in-space thrust estimation. This observation suggests the possibility that the correction may have instead worsened the ΔV execution error displacing it from near the center to the bottom-left direction in Fig. 10.

On the other hand, it is important to consider the influence of the thermal environment on the angle error observed during DV2. The thermal conditions experienced during DV2 differ from those during DV1, when the thrust estimation was performed. Given the sensitivity of the propulsion system to the thermal environment, it is possible that the angle error is related to the behavior of the propulsion system rather than structural misalignment. Further analysis is required to determine the extent to which thermal effects contribute to the observed discrepancy in the executed thrust vector.

Additionally, it is essential to recognize the potential impact of orbit determination accuracy on the reliability of the executed thrust vector. The accuracy of orbit determination is inherently subject to limitations and uncertainties, which can introduce errors into the calculated thrust vector. Therefore, the observed angle error may be influenced by the accuracy of orbit determination, further complicating the assessment of the executed thrust vector.

To further evaluate the efficacy of the estimation methods for DV1, a simulation was conducted in collaboration with the orbit and trajectory team. The simulation assumed no trajectory correction maneuvers (TCM) were performed during a specific

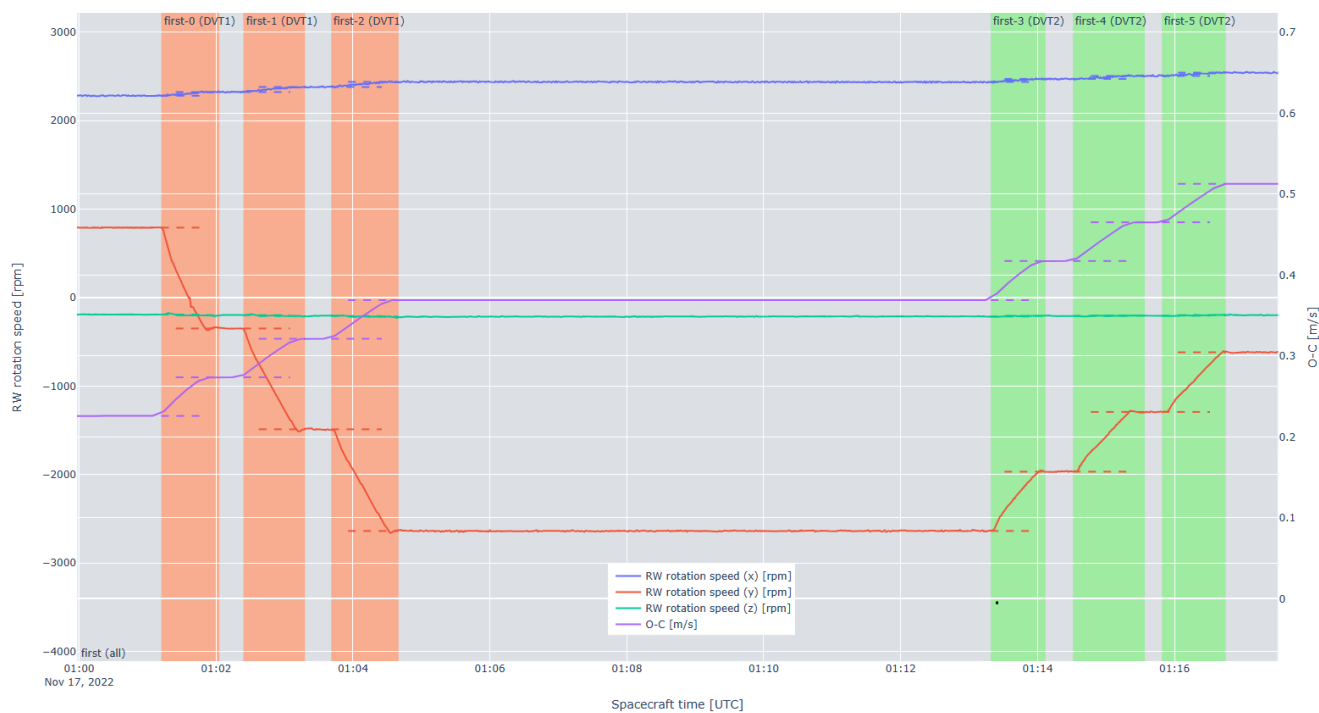


Fig. 7. Reaction wheel rotation speeds and Doppler measurements (residual o-c) at the first attitude during EQUULEUS operation. Each colored rectangle correspond to a single-thruster activation as indicated by the corresponding name tags.

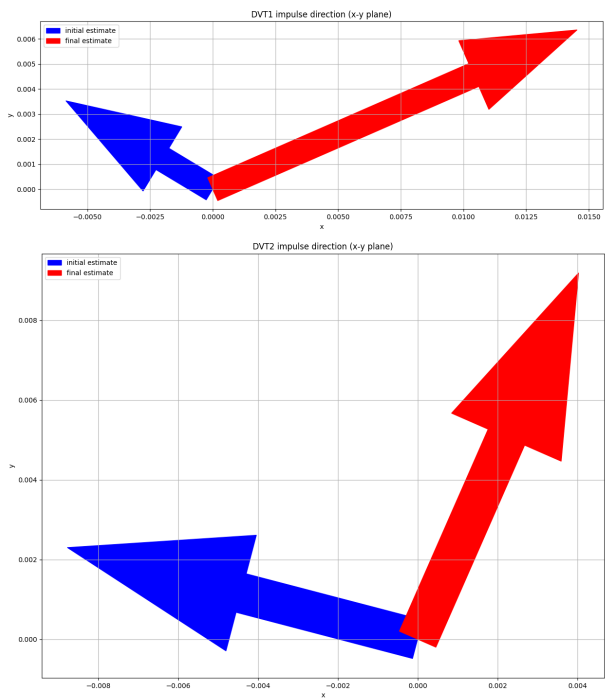


Fig. 8. Comparison of impulse unit direction components in x-y plane. Initial estimation with the previous algorithm is shown in blue, while final estimation with the proposed algorithm is shown in red.

period, from DV1 to November 23rd, 2022. The distance error, which represents the disparity between the desired and executed trajectories of the spacecraft, was calculated and compared for both the initial estimation and the proposed algorithm.

The simulation results revealed a significant improvement in the accuracy of the estimated trajectory when using the proposed algorithm compared to the initial estimation. Specifi-

cally, the error associated with the initial estimation was found to be over 70 km larger than the error obtained with the proposed algorithm. These findings are presented in table 5, which summarizes the distance error for each case. These results demonstrate the efficacy of the proposed algorithm in improving the overall trajectory accuracy of the spacecraft compared to the previous one.

Table 5. Distance error based on orbit propagation from DV1.

	Distance error [km]
Initial Estimate	182.59
Final Estimate	109.44

10. Conclusion

A novel in-space thrust estimation algorithm for the EQUULEUS spacecraft has been presented in this paper. The pro-

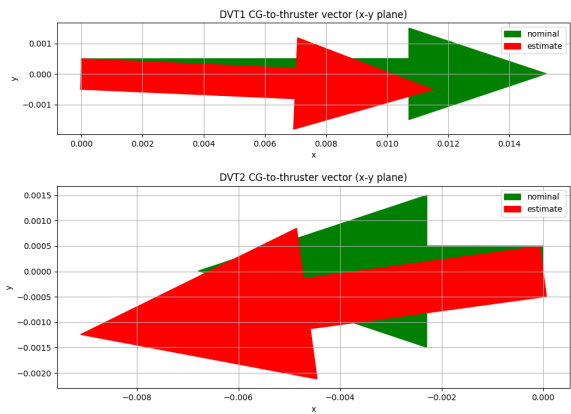


Fig. 9. Comparison between nominal and estimated center of gravity to thruster vector x-y components for both thrusters during operation.

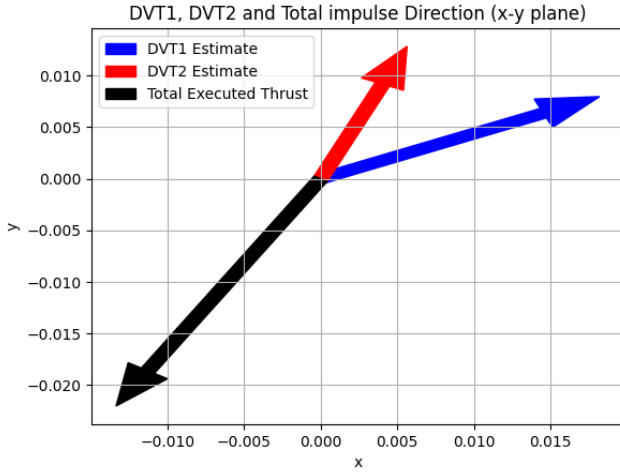


Fig. 10. Comparison of estimated thrust direction for each thruster and total executed thrust during second ΔV maneuver.

posed observation strategy and attitude selection methodologies have demonstrated feasibility and promise in advancing thrust vector estimation. Numerical simulations have validated the algorithm, showcasing its superior performance compared to the previous approach in accurately estimating the true thrust direction. Furthermore, the effectiveness of in-space thrust vector estimation during spacecraft operation and the impact of corrections made to each thruster on the overall ΔV execution error were also presented and discussed.

While the algorithm's efficacy has been established, challenges in evaluating its performance and understanding the impact of thruster corrections have been identified. The lack of information regarding the executed thrust direction during DV1, coupled with the low reliability of each thruster's executed thrust vector, underscore the complexities involved in accurate estimation. Furthermore, the influence of the thermal environment and the accuracy of orbit determination contribute to the intricacies of assessing the executed thrust vector.

Addressing these challenges and further refining in-space thrust vector estimation are vital for enhancing the accuracy and reliability of spacecraft operations. Future work may involve conducting comprehensive experiments, mitigating thermal effects, improving individual thruster reliability, refining thrust direction estimation, and enhancing orbit determination accuracy. By overcoming these obstacles, advancements in in-space thrust vector estimation can be achieved, leading to more precise spacecraft trajectory control and improved mission outcomes.

Acknowledgments

The work described in this paper is part of a bigger project: EQUULEUS. For that reason, the authors are grateful to all members of the EQUULEUS Team for their continued effort and support prior to and over the course of the mission.

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Appendix

Consider A as defined in Eqs. (11) and (12). Remember that for any matrix $B \in M_{n \times m}$ with real valued inputs, we have that $\text{rank}(B^T B) = \text{rank}(B)$. Therefore, for $n = 2$ it is true that

$$\begin{aligned} \text{rank}(A^T A) &= \text{rank} \begin{bmatrix} k_1^{(1)} & k_2^{(1)} & k_3^{(1)} & 0 & 0 \\ 0 & -\bar{x}_6 & \bar{x}_5 & 0 & \bar{x}_3 \\ \bar{x}_6 & 0 & -\bar{x}_4 & -\bar{x}_3 & 0 \\ -\bar{x}_5 & \bar{x}_4 & 0 & \bar{x}_5 & -\bar{x}_1 \\ k_1^{(2)} & k_2^{(2)} & k_3^{(2)} & 0 & 0 \\ 0 & -\bar{x}_6 & \bar{x}_5 & 0 & \bar{x}_3 \\ \bar{x}_6 & 0 & -\bar{x}_4 & -\bar{x}_3 & 0 \\ -\bar{x}_5 & \bar{x}_4 & 0 & \bar{x}_5 & -\bar{x}_1 \end{bmatrix}, \\ &= \text{rank} \begin{bmatrix} k_1^{(1)} & k_2^{(1)} & k_3^{(1)} & 0 & 0 \\ k_1^{(2)} & k_2^{(2)} & k_3^{(2)} & 0 & 0 \\ 0 & -\bar{x}_6 & \bar{x}_5 & 0 & \bar{x}_3 \\ \bar{x}_6 & 0 & -\bar{x}_4 & -\bar{x}_3 & 0 \\ -\bar{x}_5 & \bar{x}_4 & 0 & \bar{x}_5 & -\bar{x}_1 \end{bmatrix}. \end{aligned}$$

Let R_i represent the i -th row of the considered matrix. With the elimination $R_5 \rightarrow R_5 + \frac{\bar{x}_5}{\bar{x}_3} R_4 + \frac{\bar{x}_1}{\bar{x}_3} R_3$ we obtain:

$$\begin{aligned} \text{rank}(A^T A) &= \\ \text{rank} \begin{bmatrix} k_1^{(1)} & k_2^{(1)} & k_3^{(1)} & 0 & 0 \\ k_1^{(2)} & k_2^{(2)} & k_3^{(2)} & 0 & 0 \\ 0 & -\bar{x}_6 & \bar{x}_5 & 0 & \bar{x}_3 \\ \bar{x}_6 & 0 & -\bar{x}_4 & -\bar{x}_3 & 0 \\ -\bar{x}_5 + \frac{\bar{x}_5}{\bar{x}_3} \bar{x}_6 & \bar{x}_4 - \frac{\bar{x}_1}{\bar{x}_3} \bar{x}_6 & \frac{\bar{x}_5 \bar{x}_1 - \bar{x}_4 \bar{x}_5}{\bar{x}_3} & 0 & 0 \end{bmatrix}. \end{aligned}$$

Finally, by substituting the nominal values considered in Eq. (9) we have that

$$\text{rank}(A^T A) = \text{rank} \begin{bmatrix} k_1^{(1)} & k_2^{(1)} & k_3^{(1)} & 0 & 0 \\ k_1^{(2)} & k_2^{(2)} & k_3^{(2)} & 0 & 0 \\ 0 & -\bar{x}_6 & 0 & 0 & \bar{\Delta} p_{DV} \\ \bar{x}_6 & 0 & \mp d & -\bar{\Delta} p_{DV} & 0 \\ 0 & \pm d & 0 & 0 & 0 \end{bmatrix}.$$