

In-space Thrust Estimation for Water Resistojet Propulsion System in EQUULEUS

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This study shows algorithms of in-space thrust estimation operation in EQUULEUS. Due to the strict timeline and limited communication opportunities before the first lunar gravity assist in EQUULEUS mission, there are chances that the thrust force cannot be monitored at the end of the first deltaV operation and automatic thrust deactivation is necessary. With this background, the in-space thrust force is estimated based on the Doppler measurement of line-of-sight velocity and the measurement of angular momentum of the spacecraft. A statistics-based approach is taken so that a flexible operation design is possible. The effects of the amount of observation data and change in center of mass location during operation on estimation performance are also examined. The algorithms are demonstrated with numerical simulations.

Key Words: In-space Thrust Estimation, Delta-V, Attitude Control, CubeSat, EQUULEUS

Nomenclature

n	: number of thrust activation times
Δt	: period of a single activation of thruster, s
f	: true thrust, N
$\bar{F}$	: mean of f over a single activation, N
$\hat{F}$	: estimation of F , N
$\bar{\bar{F}}$	: expected value of F , N
$\hat{\bar{F}}$	: estimation of $\bar{F}$, N
r_{sc}	: unit vector to spacecraft from Earth
m_{sc}	: mass of spacecraft, kg
k	: vector to spacecraft with norm of m_{sc}^{-1}
r	: vector to DVT from center of mass, kg
L	: angular momentum, Nms
V_{los}	: line-of-sight velocity, m/s
v_L	: observation error of L , Nms
$v_{V_{los}}$	: observation error of V_{los} , m/s
$s_{\bar{F}}^2$	: unbiased estimator of variance of $\bar{F}$
I_{sp}	: specific impulse of DVT, s
C	: mass flow rate, kg/s
g_n	: standard gravity, m/s ²
Subscripts	
i	: index of thrust activation
0	: initial

1. Introduction

Although sailing spacecraft in space generally requires a substantial amount of delta-V, the fuel consumption can be considerably reduced if the spacecraft can accurately make the use of gravity assists by celestial bodies. This technology of precise maneuvering will be demonstrated in the Sun-Earth-Moon system in the EQUULEUS (EQUilibrium Lunar-Earth point 6U Spacecraft) project, together with the water resistojet propulsion system.¹⁻³⁾ EQUULEUS is a deep space CubeSat being developed by the

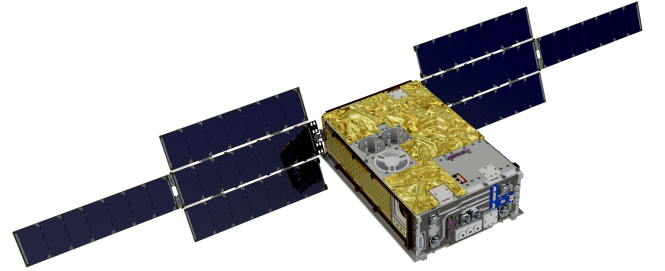


Fig. 1. External view of EQUULEUS. The body size (excluding the solar array paddles) is about 10×20×30 cm and the weight is about 11.5 kg.

University of Tokyo and the Japan Aerospace Exploration Agency (JAXA), expecting its launch in 2020 by NASA's Space Launch System.

EQUULEUS is accelerated by the water resistojet thrusters to realize lunar gravity assist maneuver and cruises to its destination, the second Earth-Moon Lagrange point (EML-2).²⁾ This first delta-V operation (called "DV1") followed by the lunar swing-by is one of the most critical operations because the failure of DV1 makes it difficult for EQUULEUS to reach its destination, EML-2. This is why we plan to perform a thrust estimate operation in space before DV1.

There are several factors that make accurate thrust estimation particularly important and challenging. One factor is the limited communication opportunities by the end of DV1 due to the limited availability of the ground stations, which will be explained in Section 2. Another factor is that no orbital transfer by water resistojet thrusters has not yet been demonstrated in space and we have little knowledge of the thruster's performance in a real mission.

This paper presents an estimation methodology for a low-thrust propulsion system with periodical output, which requires a shorter period of operation and is ap-

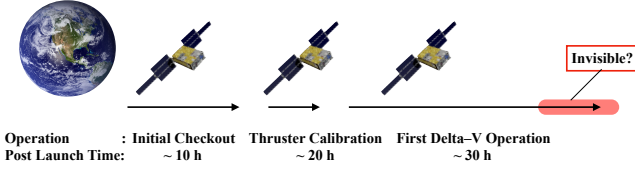


Fig. 2. Timeline for thrust estimation and first delta-V operation (DV1). There is a possibility that DV1 need to be terminated without ground communication.

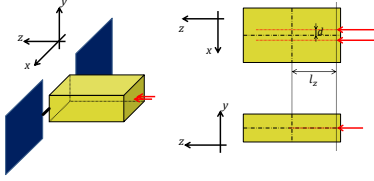


Fig. 3. Layout of delta-V thrusters (DVTs) in EQUULEUS. Thrust vectors are shown in red arrows.

plicable to a strict mission timeline.

2. Nominal Initial Operation: from Checkout to DV1

The first and most critical operation in EQUULEUS mission is the first delta-V operation, or DV1. What makes DV1 challenging is the strict timeline and limited communication opportunities before DV1. Figure 2 shows the initial operation timeline up to DV1. DV1 in the nominal schedule is initiated within only about 30 hours after the separation and by then initial checkout and thrust calibration have to be completed. Besides, there are chances that no ground station capable of deep space missions is available at the end of DV1, which means full monitoring of thrust force by Doppler measurement like other missions may not be possible.⁴⁾ In such a case, thrusters have to be automatically deactivated at the right timing with the right resulting delta-V and thus accurate knowledge of thrust force is necessary in advance. This is why in-space thrust estimation is planned to be carried out during the thrust calibration. Normally, the thrust force in space can be estimated precisely by taking the two long data arcs before and after thrust execution. This method, however, cannot be used as it is too time-consuming for this timeline. Instead, we plan to perform a shorter term in-space thrust estimation based on a statistical approach as described in Section 4.

3. Water Resistojet Delta-V Thruster

The water resistojet propulsion system in EQUULEUS is called AQUARIUS (AQUA ResIstojet propUlsion System) and Table 1 shows the specification of AQUARIUS.³⁾ It is a low-thrust propulsion system and has six thrusters in total; four reaction control system (RCS) thrusters for momentum dumping of reaction wheels and two delta-V thrusters for the orbital transfer. The latter two thrusters are referenced as DVTs or DV thrusters. The two DV thrusters are located symmetrically across the body frame z-axis and their thrust vectors are parallel to the z-axis (Fig. 3).

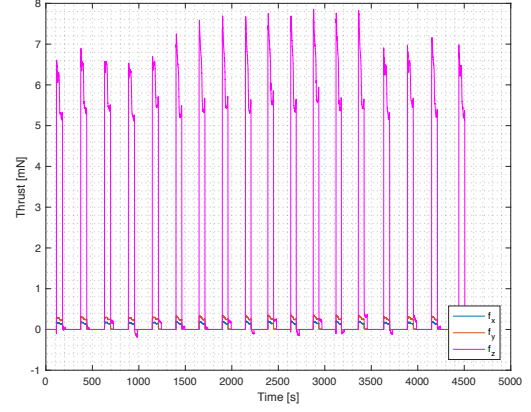


Fig. 4. Thrust response simulated based on experiment. The simulated thrust vector is tilted a little on purpose and thus x and y component of the thrust vector has a nonzero value.

The basic mechanism of DVT can be explained as analogous to a kettle releasing vaporized water; each activation of DVT makes the chamber release the preheated steam that originates from the fixed amount of water injected every time of activation. During DV1, those two DVTs are alternately activated and deactivated providing periodic thrust output. At the operation for in-space thrust estimation, however, the two DVTs are tested individually. Figure 4 shows the thrust force profile over the repeated activation of a single DVT, which is based on ground experiment and modified a little so that the simulated thrust vector is tilted a little with respect to the nominal direction for validation purpose. The value to be estimated is the mean thrust vector over all activations, $\bar{F}$, defined as

$$\bar{F} = E[F_i] = E\left[\frac{\int_{t_{0,i}}^{t_{0,i}+\Delta t_i} f_i(t)dt}{\Delta t_i}\right]. \quad (1)$$

4. Thrust Estimation and Delta-V Prediction

The thrust estimation for DV1 is planned to have 3 stages.

1. Individual Estimation: Mean thrust for each activation, $F(i)$, is estimated.
2. System Estimation: Overall mean thrust, $\bar{F}$, and its variance is estimated.
3. Prediction: Resulting Delta-V is estimated for DV1.

In the following sections each stage is expounded in details.

Table 1. Specifications of AQUARIUS.³⁾

AQUARIUS	
Propellant	Water(< 100 kPa)
Thrust	4.0 mN
Specific impulse	70 s
Power consumption	< 20 W
Number of delta-V thrusters	4
Number of RCS thrusters	2
Volume (CubeSat unit)	2.0 U

4.1. Individual estimation of mean thrust

To estimate the in-space thrust, we use two kinds of observables; the spacecraft angular momentum (Eq. (2)) and the line-of-sight (direction from the Earth to the spacecraft) velocity of the spacecraft (Eq. (3)). The angular momentum is measured inside the spacecraft to be down-linked and the line-of-sight velocity is measured by the Doppler effect from the ground during the thrust execution. Note that thermal condition is critical to the performance of water resistojet propulsion system and thrust estimation operation needs to be performed in the same attitude as DV1 with respect to the Sun, which leaves one degree of freedom in the rotation angle about Sun direction in the body frame and $k = r_{sc}/m_{sc}$ has to be defined to meet this condition.

$$\Delta L_i = \int_{t_{0,i}}^{t_{0,i}+\Delta t_i} r \times f_i(t) dt + v_{L,i} \quad (2)$$

$$\Delta V_{los,i} = \int_{t_{0,i}}^{t_{0,i}+\Delta t_i} k \cdot f_i(t) dt + v_{V_{los},i} \quad (3)$$

The Eqs. (2) and (3) can be combined into the equation of observation (Eq. (4)).

$$z_i = Ax_i + w_i \quad (4)$$

, where

$$z_i = \begin{bmatrix} \Delta L_i / \Delta t_i \\ \Delta V_{los,i} / \Delta t_i \end{bmatrix}, A = \begin{bmatrix} 0 & -r_z & r_y \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \\ k_x & k_y & k_z \end{bmatrix},$$

$$x_i = \frac{\int_{t_{0,i}}^{t_{0,i}+\Delta t_i} f_i(t) dt}{\Delta t_i}, \quad w_i = \begin{bmatrix} v_{L,i} / \Delta t_i \\ v_{V_{los},i} / \Delta t_i \end{bmatrix}.$$

Estimation of $F(i)$ is given by the Least Square Solution as shown in Eq. (5).⁵⁾

$$\hat{F}_i = (A^T A)^{-1} A^T z_i \quad (5)$$

4.2. System estimation of mean thrust

Estimation of overall mean thrust, $\bar{F}$, can be rephrased with statistical terms as the estimation of population mean, $\bar{F}$, with given samples, $\hat{F}_i$. Thus, $\bar{F}$ and an unbiased estimator of variance of $\bar{F}$ is calculated as Eqs. (6) and (7).⁶⁾

$$\hat{\bar{F}} = \frac{\sum_{i=1}^n \hat{F}_i}{n} \quad (6)$$

$$s_{\bar{F}}^2 = \frac{1}{n-1} \sum_{i=1}^n \left(\hat{F}_i - \hat{\bar{F}} \right)^2 \quad (7)$$

Observation based confidence interval of $\bar{F}$ can be calculated as

$$CI[\bar{F}]_{obs} = \hat{\bar{F}} \pm t_{\frac{\alpha}{2}}(n-1) \sqrt{\frac{s_{\bar{F}}^2}{n}} \quad (8)$$

, where $t_{\frac{\alpha}{2}}(n-1)$ is the t-statistic with DOF of $n-1$ based on a certain confidence level of α .⁶⁾

To estimate accurate $\bar{F}$, it is important to have enough number of samples as seen in Eq. (8). This means DVT should be repeatedly activated for enough number of times in the estimation operation. Given the covariance of thruster's reproduction accuracy, Σ_{F_i} and covariance of observation error, Σ_w , the covariance of $\hat{F}_i$, can be calculated as

$$\begin{aligned} \Sigma_{\hat{F}_i} &= E[(\hat{F}_i - \bar{F})(\hat{F}_i - \bar{F})^T] \\ &= \Sigma_{F_i} + (A^T A)^{-1} A^T \Sigma_w A (A^T A)^{-1} \\ &= \Sigma_{F_i} + \Sigma_{obs}. \end{aligned} \quad (9)$$

By the Central Limit Theorem, it is known that the variance of $\hat{F}_i$ becomes $\Sigma_{\hat{F}}/n$ and decreases as the number of samples, n , increases. So, if Σ_{F_i} and Σ_w is well known you can also calculate model based confidence interval of $\bar{F}$ as

$$CI[\bar{F}]_{model} = \hat{\bar{F}} \pm Z_{\frac{1-\alpha}{2}} \sqrt{\frac{\text{diag}(\Sigma_{F(i)} + \Sigma_{obs})}{n}} \quad (10)$$

, where Z is the z-score with a confidence level of α .⁶⁾ So, the operator can also determine the number of activation n for estimation operation with the outcome of Eq. (10).

4.3. Delta-V Prediction

Delta-V at DV1 can be predicted by the Tsiolkovsky rocket equation (Eq. (12)) with the estimation of the specific impulse (Eq. (11)).⁷⁾ Confidence interval of delta-V can be calculated by substituting $\hat{\bar{F}}$ for $CI[\bar{F}]$ in Eq. (11).

$$\hat{I}_{sp} = \frac{\hat{\bar{F}}}{C g_n} \quad (11)$$

$$\Delta \hat{V} = \hat{I}_{sp} g_n \ln \frac{m_0}{m_0 - C t} \quad (12)$$

5. Evaluation of Varying Center of Mass

There are some factors that potentially affect estimation performance in a real mission. Among the effects of those factors, this section examines the effect of change in the vector from spacecraft center of mass to a DVT, which will vary over the transformation of the bladder fuel tank due to fuel consumption. Errors in time synchronization between spacecraft and ground station are not likely to affect estimation as long as the index of thrust activation can be correctly assigned to each observation, which will not be an issue given the data profile and marginal errors in the time synchronization.

The vector from spacecraft center of mass to a DVT, r , comes into play for thrust estimation as elements of A (Eq. 5). To evaluate the effects of error in r on thrust estimation, let $A = \hat{A} + G$, where $\hat{A}$ is the nominal matrix and G is formed by substituting errors in r for r in A at Eq. 4 and 4th row of G is all zero. By substituting $A = \hat{A} + G$ in Eq. (5), we get

$$\hat{F}_i = F_i + (\hat{A}^T \hat{A})^{-1} \hat{A}^T w - (\hat{A}^T \hat{A})^{-1} \hat{A}^T G F_i. \quad (13)$$

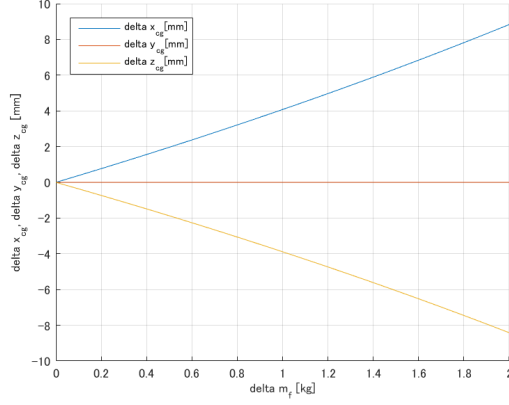


Fig. 5. Transition of center of mass location due to fuel consumption

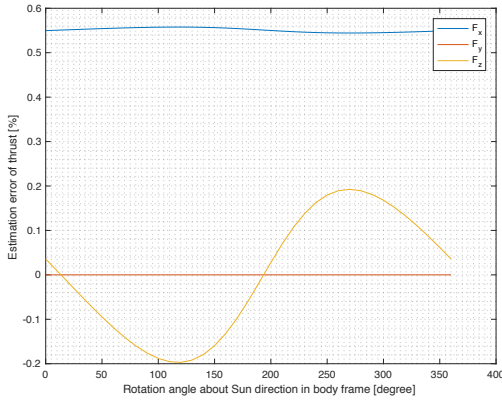


Fig. 6. Error effects of the vector from spacecraft center of mass to DVT; 1mm error in x

Taking expectation of Eq. (13) gives

$$E[\hat{F}(i)] = \bar{F} - (\hat{A}^T \hat{A})^{-1} \hat{A}^T G \bar{F}. \quad (14)$$

The magnitude of the second term in Eq. (14) is the effects of error in r on the thrust estimation.

Figure 5 shows the change in the center of mass over fuel consumption and Fig. 6 shows the effects of the error of 1 mm in body x coordinate over spacecraft attitude. The fuel consumption by the in-space thrust estimation will be marginal and should not exceed 0.2 kg. You can see there is little effect on the estimation performance. Note that y coordinate of the center of mass will be constant and change in z coordinate will hardly affect estimation performance because the nominal thrust vector is parallel to the z-axis of the body frame.

6. Numerical Simulation

In this section, the aforementioned examination and estimation are presented based on a simulated operation of in-space thrust estimation. This simulation uses the thrust profile of Fig. 4 as the DVT output.

6.1. Model based CI for operation assessment

Model based confidence interval is calculated with Eq. (10) to assess the effect of number of activation times during in-space thrust estimation on the expected per-

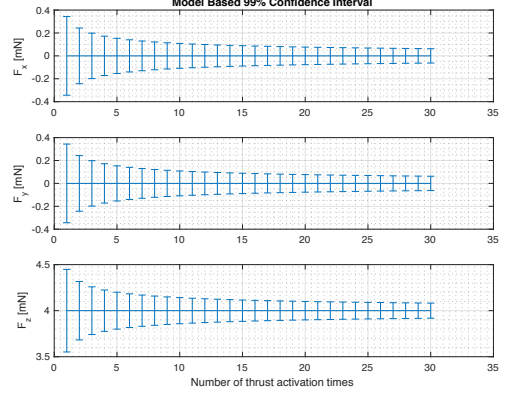


Fig. 7. Model based confidence interval (99%) of mean thrust

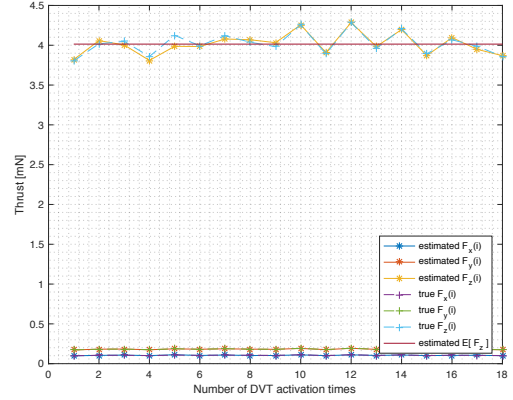


Fig. 8. Estimation results

formance of estimation. To calculate $\Sigma_{F_i} + \Sigma_w$, accuracy of thrust reproduction is approximated by 90% and standard deviation of observation errors is approximated by 3.3×10^{-6} Nms and 0.5×10^{-3} m/s for angular momentum and line-of-sight velocity respectively. Uncertainty is approximated with Gaussian distribution. Here, the estimated nominal DVT thrust is set $\bar{F} = [0 \ 0 \ 4]^T$ mN. The results are shown in Fig. 7, based on which we set the number of activation times for following simulation 18.

6.2. Estimation of mean thrust

Figure. 8 shows the sequence of the estimated thrust vectors for each activation and its true value. You can see the thrust vectors are estimated well and the simulated deviation from the body frame z-axis, which is the nominal thrust direction, is detected as well. This figure also helps ground operators grasp the general profile of thrust force, which is important in a real operation in order to detect any unexpected profile due to system malfunction.

6.3. Delta-V Prediction

The results of Delta-V prediction together with the 99% observation based confidence interval is shown in Fig. 9. In real operation, DV1 operation is planned to be scheduled with the help of this figure with real values.

7. Conclusion

This study showed the statistical based algorithms and simulation results of in-space thrust force estimation designed for the strict timeline and limited communication

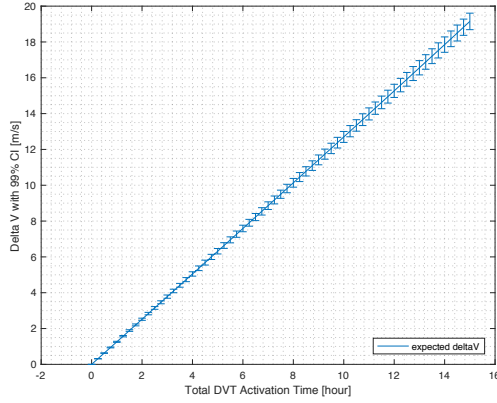


Fig. 9. Delta-V prediction and 99% observation based confidence interval

opportunities of the initial operation of EQUULEUS. In-space thrust estimation by the measurement of angular momentum and the Doppler measurement of line-of-sight velocity revealed its feasibility in EQUULEUS mission. This algorithm assumes the change of line-of-sight velocity and angular momentum due to each activation of the propulsion system can be indirectly observed with noise and can be correlated with each other without confusion. Future work includes designing other supplementary estimation algorithms so that mission designers can deal with the situations where even the above assumptions do not hold. An example of those supplementary estimation algorithms will be the one that utilizes telemetry points of temperature and pressure in addition to angular momentum, which ultimately makes thrust estimation possible

even independent from the ground observation and communication.

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